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Conformally invariant pomeron interaction in perturbative QCD with large N_c

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Abstract. An effective non-local quantum field theory is constructed, which describes the interaction of pomerons in high-colored QCD. The theory includes both splitting and merging triple pomeron vertexes and diagrams with pomeronic loops. The Schwinger–Dyson equations for the 'physical' pomeron are written. Conformal invariance allows one to reduce the theory to the old-fashioned Gribov pomeron theory with an infinite number of pomerons, one of which is supercritical.

1 Introduction

The high-energy behavior in QCD with a large number of colors N_c is described by the exchange of hard pomerons, which split and merge by a triple pomeron vertex. Exchange of colored objects (single gluons) is damped by a factor $1/N_c^2$. In the interaction with heavy nuclei, the leading contribution comes from diagrams without pomeronic loops (tree diagrams). Summation of these is achieved by a closed equation for DIS (the BK equation [1–4]) or a closed pair of equations for nucleus–nucleus scattering [5–7]. Some estimates of the contribution from loops were made in [8, 9]. To take into account the contribution from pomeronic loops in a consistent manner one has to consider the effective pomeron field theory introduced in [5–7] as a full-fledged quantum theory. The present study is devoted to this aim.

Note that pomeronic loops have lately also been actively studied in the framework of the color dipole picture in the so-called JIMWLK approach (see e.g. [10] and references therein). There, the evolution in rapidity of a state, considered as a functional of the gluon field, is governed by a certain Hamiltonian made of the field and functional derivatives in the field. Taking loops into account leads to a Hamiltonian containing functional derivatives up to the fourth order [11]. In our approach a quantum Hamiltonian can also be introduced. However, it contains functional derivatives only up to the second order and so is considerably simpler than in the JIMWLK approach.

In this study we restricted ourselves to a rather formal treatment of the perturbative diagrams for the pomeron interaction with or without loops. We construct the relevant Schwinger–Dyson equations for the full pomeron Green function and also discuss their conformal (Möbius) invariance, which hopefully may simplify their analysis. In fact we are not very optimistic about a realistic calculation of the amplitudes with pomeronic loops included. This problem presented enormous difficulties already for the much simpler old-fashioned Gribov local supercritical pomeron model, so that even its internal inconsistency was claimed [12] (see also [13] for a discussion of this inconsistency). Possibly the more complicated structure of the BFKL pomeron and its interaction may overcome these old troubles. However, in the present study this problem is not touched but left for future investigations.

2 Effective action and diagrams

2.1 Effective action

Our main tool will be the non-forward BFKL Green function as a function of gluon coordinates and rapidities, which satisfies the equation

$$\left(\frac{\partial}{\partial y}+H\right) g\left(y-y';r_1,r_2;r_1',r_2'\right) = \delta(y-y')\nabla_1^{-2}\nabla_2^{-2}\delta^2(r_{11'})\delta^2(r_{22'})\,, \qquad (1)$$

where $r_{11'}=r_1-r_1'$ etc. and H is the BFKL Hamiltonian [14, 15]:

$$H=\frac{\bar\alpha}{2}\left(\ln p_1^2+\ln p_2^2+\frac{1}{p_1^2}\ln r_{12}^2p_1^2+\frac{1}{p_2^2}\ln r_{12}^2p_2^2-4\psi(1)\right), \qquad (2)$$

p_1 and p_2 are the reggeized gluons transverse momenta, and $\bar\alpha=\alpha_{\rm s}N_c/\pi$. To economize on notation, in the following we shall denote as z the point in the space formed by rapidity y and two transverse vectors r_1, r_2:

$$z=\{y,r_1,r_2\}=\{y,\varrho\}\,.$$

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In this notation we write $g(y-y';r_1,r_2;r_1',r_2')=g(z,z')$. The Green function $g(z,z')$ is invariant under conformal (Möbius) transformations of coordinates r_1 and r_2. In complex notation, $r=x=\mathrm{i}y$, these transformations are

$$r'=\frac{\alpha r+\beta}{\gamma r+\delta}\,,\quad r'^{*}=\frac{\alpha r^{*}+\beta}{\gamma r^{*}+\delta}\,,$$

with $\alpha\delta-\beta\gamma=1$. The inverse function $g^{-1}(z,z')$ defined by

$$\int \mathrm{d}z''g^{-1}(z,z'')g(z'',z')=\delta(z-z')\,, \tag{3}$$

where $\mathrm{d}z=\mathrm{d}y\,\mathrm{d}^2r_1\,\mathrm{d}^2r_2\equiv\mathrm{d}y\,\mathrm{d}\varrho$, and $\delta(z)=\delta(y)\times\delta^2(r_1)\delta^2(r_2)\equiv\delta(y)\delta\varrho$, is however not conformally invariant, since the measure is not conformally invariant. It is not difficult to construct a conformally invariant inverse g^{-1}_{inv} using the invariant measure:

$$\mathrm{d}\tau=\frac{\mathrm{d}^2r_1\,\mathrm{d}^2r_2}{r_{12}^4}\,. \tag{4}$$

Then we can rewrite (3) as

$$\int \mathrm{d}y''\,\mathrm{d}\tau''r_{12}^4g^{-1}(z,z'')r_{12}''^{\,4}g(z'',z')=r_{12}^4\delta(z-z')\,. \tag{5}$$

This shows that the conformally invariant function is

$$g^{-1}_{\mathrm{inv}}(z,z')=r_{12}^4g^{-1}(z,z')r_{12}'^{\,4}\,. \tag{6}$$

Now we pass to constructing the effective non-local quantum field theory, which is to describe propagation of free pomerons and their triple interaction. It has to generate all diagrams built from the pomeron propagator, which is the Green function g, and the triple pomeron vertex introduced in [16–19]. Let $\varphi_B(z)$ and $\varphi_A(z)$ be two bilocal fields for the incoming and outgoing pomerons. To reproduce the correct propagator the free action S_0 has to be

$$S_0=\int \mathrm{d}z\,\mathrm{d}z'\varphi_A(z)g^{-1}(z,z')\varphi_B(z)\,. \tag{7}$$

We introduce interaction with external sources as

$$S_{\mathrm{E}}=-\int \mathrm{d}z\varphi_A(z)J_B(z)+(A\leftrightarrow B)\,. \tag{8}$$

Here the sources for the projectile A and target B are different from zero at rapidities $y=Y$ and $y=0$, respectively:

$$J_A(z)=\bar J_A(\varrho)\delta(y-Y)\,,\quad J_B(z)=\bar J_B(\varrho)\delta(y)\,. \tag{9}$$

Finally the interaction to reproduce the 3P vertex at large N_c has to be taken in the form

$$S_{\mathrm{I}}=\frac{2\alpha_{\mathrm{s}}^2N_c}{\pi}\int \mathrm{d}y\frac{\mathrm{d}^2r_1\,\mathrm{d}^2r_2\,\mathrm{d}^2r_3}{r_{12}^2r_{23}^2r_{31}^2}\varphi_B(z_1)\varphi_B(z_2)L_{12}\varphi_A(z_3)+(A\leftrightarrow B)\,, \tag{10}$$

where $z_1=\{y,r_2,r_3\}$, $z_2=\{y,r_3,r_1\}$, $z_3=\{y,r_1,r_2\}$ and the conformally invariant operator L_{12} is

$$L_{12}=r_{12}^4\nabla_1^2\nabla_2^2\,. \tag{11}$$

Note that the form (10) assumes the fields to be symmetric in the two space points r_1 and r_2. In fact the symmetry properties of the fields are determined by symmetry properties of the external sources. We assume them to be symmetric under the interchange $r_1\leftrightarrow r_2$.

This action leads to BFKL pomeron diagrams with the standard 3P interaction in the presence of an external field. Note that the signs of different building blocks of the diagrams are somewhat different from the standard ones: the propagator g and external sources enter with a minus sign as a consequence of the choice of signs in the action. This latter corresponds to the desire to make the interaction real and not pure imaginary as in the original Gribov reggeon field theory.

In the absence of the external sources this action is explicitly conformally invariant provided φ_B and φ_A are invariant. Indeed the free part can be rewritten as

$$S_0=\int \mathrm{d}y\,\mathrm{d}y'\,\mathrm{d}\tau\,\mathrm{d}\tau'\varphi_A(z)r_{12}^4g^{-1}(z,z')r_{12}'^{\,4}\varphi_B(z) \tag{12}$$

and the interaction part as

$$S_{\mathrm{I}}=\frac{2\alpha_{\mathrm{s}}^2N_c}{\pi}\int \mathrm{d}y\,\mathrm{d}y'\,\mathrm{d}y''\,\mathrm{d}\tau\,\mathrm{d}\tau'\,\mathrm{d}\tau''\varphi_B(z')\varphi_B(z'')\times\gamma(z',z''|z)L_{12}\varphi_A(z)+(A\leftrightarrow B)\,, \tag{13}$$

where $\gamma(z',z''|z)$ is the bare interaction vertex for the incoming pomeron at z and two outgoing pomerons at z' and z'':

$$\gamma(z',z''|z)=\delta(y-y')\delta(y-y'')\delta^2(r_{12'})\delta^2(r_{1'2''})\delta^2(r_{1''2})\times r_{12}^2r_{1'2'}^2r_{1''2''}^2\,. \tag{14}$$

This is a conformally invariant function. Since both $r_{12}^4g^{-1}(z,z')r_{12}'^{\,4}$ and $\gamma(z',z''|z)$ are conformally invariant, so is the action S_0+S_{I}. Of course, in physically relevant cases conformal invariance is always broken by the external sources, which also introduce a mass scale into the theory. As a result the contribution from any Feynman diagram without external sources is conformally invariant, which becomes explicit if one uses the invariant integration measure $\mathrm{d}z_{\mathrm{inv}}=\mathrm{d}y\mathrm{d}\tau$ and the invariant interaction vertex γ given by (14).

Finally, note that from (1) it follows that as an operator in the z-space

$$g^{-1}(z,z')=\nabla_\varrho^2\left(\frac{\partial}{\partial y}+H\right)\delta(z-z')\,. \tag{15}$$

Here we use the notation

$$\nabla_\varrho^2=\nabla_1^2\nabla_2^2\,. \tag{16}$$

Note that the operator ∇_ϱ^2H is symmetric:

$$\nabla_\varrho^2H=\frac{\bar\alpha}{2}\left(p_1^2p_2^2\ln\left(p_1^2p_2^2\right)+p_2^2\ln r_{12}^2p_1^2+p_1^2\ln r_{12}^2p_2^2-4\psi(1)p_1^2p_2^2\right). \tag{17}$$

As a result S_0 can be written directly in terms of the BFKL Hamiltonian:

$$S_0 = \int \mathrm{d}y\,\mathrm{d}\varrho\varphi_A(y,\varrho)\nabla_\varrho^2\left(\frac{\partial}{\partial y}+H\right)\varphi_B(y,\varrho) = \int \mathrm{d}y\,\mathrm{d}\varrho\varphi_B(y,\varrho)\nabla_\varrho^2\left(-\frac{\partial}{\partial y}+H\right)\varphi_A(y,\varrho)\,. \tag{18}$$

In this form the symmetry between the projectile and target is made explicit: it has to be accompanied by changing $y \to -y$.

Using for H the representation [20]

$$Hf(r_1,r_2) = \frac{\bar\alpha}{2\pi}\int \frac{\mathrm{d}^2r_3 r_{12}^2}{r_{13}^2 r_{23}^2}(f(r_1,r_2)-f(r_1,r_3)-f(r_2,r_3))\,, \tag{19}$$

we can rewrite the free part of the action in a more explicit form:

$$S_0 = \int \mathrm{d}y\,\mathrm{d}^2r_1\,\mathrm{d}^2r_2\varphi_A(y,r_1,r_2)\nabla_\varrho^2 \times\left\{\frac{\partial}{\partial y}\varphi_B(y,r_1,r_2)+\frac{\bar\alpha}{2\pi}\int\frac{\mathrm{d}^2r_3 r_{12}^2}{r_{13}^2r_{23}^2} \times(\varphi_B(y,r_1,r_2)-\varphi_B(y,r_1,r_3)-\varphi_B(y,r_2,r_3))\right\}. \tag{20}$$

2.2 Diagrams and their order of magnitude

The quantum field theory described by the action $S = S_0 + S_\mathrm{I} + S_\mathrm{E}$ allows one to construct the perturbation theory by the standard technique, expressing the amplitude as a sum of Feynman diagrams. It is instructive to see orders of magnitude of different contributions in terms of two independent small parameters of the theory, $\bar\alpha = \alpha_\mathrm{s}N_c/\pi$ and $1/N_c$.

First we note that the smallness of these basic parameters allows one to neglect other contributions, apart from those appearing in our effective theory. The smallness of $\bar\alpha$ allows one to drop higher order corrections to the BFKL pomeron itself. The smallness of $1/N_\mathrm{c}$ allows one to neglect contributions from the exchange of more complicated colorless structures than the BFKL pomeron, which appear if one takes into account the full interaction between the reggeized gluons and not only its leading part in $1/N_c$ [16, 17]. It also allows one to neglect higher order transition vertexes from m to n pomerons [19].

In the remaining effective theory each triple interaction contributes $\bar\alpha^2/N_c$. The final estimate depends on the magnitude of the external source. If we treat it perturbatively to be consistent with the whole approach, then it should correspond to the quark–antiquark loop, from which we extract its order, which is $\bar\alpha$. As a result, the diagram with l_E external lines (sources) and L loops has the order

$$\bar\alpha^{l_\mathrm{E}}\left(\frac{\bar\alpha^2}{N_c}\right)^{2L+l_\mathrm{E}-2}. \tag{21}$$

As one sees, for a given number of external sources, the dominant contribution comes from the tree diagrams, each loop introducing a small factor $\bar\alpha^4/N_c^2$. This implies of course that at $\bar\alpha\ln s \sim 1$ the loops are suppressed and need not be taken into account.

However, this pure counting does not take into account the growth of the pomeron propagator at large y as $\exp\Delta y$ where Δ is the BFKL intercept, nor the enhancement related to the nuclear sources with large atomic numbers. For the AA collision amplitude with the overall rapidity difference Y this changes (21) to

$$(\bar\alpha A^{1/3})^{l_\mathrm{E}}\left(\frac{\bar\alpha^2}{N_c}\right)^{2L+l_\mathrm{E}-2}\mathrm{e}^{n\Delta Y}, \tag{22}$$

where n is the maximal number of the exchanged pomerons at a given rapidity. This number depends on the topology of the diagram and generally grows with l_E and L.

So, for

$$\bar\alpha A^{1/3} \sim 1 \tag{23}$$

one has to sum all tree diagrams, and for rapidities Y such that

$$\frac{\bar\alpha^2}{N_c}\mathrm{e}^{\Delta Y} \sim 1\,, \tag{24}$$

one has to sum also at least some of the loops. For onium–onium scattering ($l_\mathrm{E}=2$) a convenient method to sum the leading contribution is to join two sums of tree diagrams starting from the projectile and target at mid-rapidity [8].

However, all these perturbative estimates are to be taken with caution. The full pomeron Green function may have an asymptotic behavior at large y very different from the bare one. In fact there is every reason to believe that the former will grow at most as a power of y, not as an exponential. Then all the above estimates will have to be reconsidered. Indications for this come from the study of a similar (but much simpler) old Regge–Gribov model for the supercritical pomeron. It was concluded that in all probability the model was inconsistent unless the physical pomeron became critical ($\Delta = 0$) or even subcritical ($\Delta < 0$) (see e.g. [12]). Unfortunately these results have not been obtained in a reliable manner up to now.

3 The pomeron Hamiltonian and operators

With the action fixed, one can easily construct a Hamiltonian formulation for the state evolution. Since the Lagrangian is of the first order in derivatives in rapidity, the Hamiltonian is just the action without the derivative terms with a minus sign and integration over y dropped:

$$\mathcal{H} = -\int \mathrm{d}\varrho\varphi_A(\varrho)\nabla_\varrho^2 H\varphi_B(\varrho) - \frac{2\alpha_\mathrm{s}^2N_c}{\pi}\int\frac{\mathrm{d}^2r_1\,\mathrm{d}^2r_2\,\mathrm{d}^2r_3}{r_{12}^2r_{23}^2r_{31}^2} \times\{\varphi_B(\varrho_{23})\varphi_B(\varrho_{31})L_{12}\varphi_A(\varrho_{12})+(A\leftrightarrow B)\}\,, \tag{25}$$

where $\varrho_{23}=\{r_2,r_3\}$ etc. To pass to the quantum theory one has to consider $\varphi_{A,B}(\varrho)$ as operators. Their commutation relation can be easily established from the form of the free Green function,

$$g(y-y';\varrho,\varrho')=-\left\langle T\left\{\varphi_B(y,\varrho)\varphi_A(y',\varrho')\right\}\right\rangle . \tag{26}$$

From (26) we conclude

$$\left(\frac{\partial}{\partial y}+H\right)\left\langle T\left\{\varphi_B(y,\varrho)\varphi_A(y',\varrho')\right\}\right\rangle = \delta(y-y')\left[\varphi_B(y,\varrho),\varphi_A(y,\varrho')\right], \tag{27}$$

where we have used the equation of motion for φ_B. Comparison with (1) gives

$$[\varphi_B(y,\varrho),\varphi_A(y,\varrho')]=-\nabla_\varrho^{-2}\delta(\varrho-\varrho') . \tag{28}$$

Thus in a representation in which $\varphi_B(\varrho)$ is diagonal and the state vector is a functional $\Psi\{\varphi_B(\varrho)\}$, the field φ_A is essentially a functional derivative:

$$\varphi_A(\varrho)=\nabla_\varrho^{-2}\frac{\delta}{\delta\varphi_B(\varrho)} . \tag{29}$$

The state with a given field $\varphi_A(\varrho)$ will be represented by a an exponential,

$$\Psi_{\varphi_A(\varrho)}(\{\varphi_B\})=\mathrm{e}^{\int \mathrm{d}\varrho\varphi_B(\varrho)\nabla_\varrho^2\varphi_A(\varrho)} . \tag{30}$$

The state vector will satisfy the evolution equation

$$\frac{\mathrm{d}\Psi}{\mathrm{d}y}=\mathcal{H}\Psi , \tag{31}$$

where in $\mathcal{H}_B$, given by (25), one has to substitute the field φ_A by functional derivatives. In this substitution, as always, the order of the operators is actually undetermined. If we put all the derivatives to the right ('normal ordering'), then explicitly

$$\begin{aligned}\mathcal{H}=&-\frac{\bar\alpha}{2\pi}\int\frac{\mathrm{d}^2r_1\,\mathrm{d}^2r_2\,\mathrm{d}^2r_3r_{12}^2}{r_{13}^2r_{23}^2}\\ &\times\left\{[\varphi_B(\varrho_{12})-\varphi_B(\varrho_{13})-\varphi_B(\varrho_{23})]\frac{\delta}{\delta\varphi_B(\varrho_{12})}\right.\\ &-4\pi\alpha_{\mathrm{s}}\left[\varphi_B(\varrho_{13})\varphi_B(\varrho_{23})\frac{\delta}{\delta\varphi_B(\varrho_{12})}\right.\\ &\left.\left.+L_{12}\varphi_B(\varrho_{12})\left(\nabla_\varrho^{-2}\frac{\delta}{\delta\varphi_B(\varrho_{13})}\right)\left(\nabla_\varrho^{-2}\frac{\delta}{\delta\varphi_B(\varrho_{23})}\right)\right]\right\} .\end{aligned} \tag{32}$$

One can pass from this squttarget representation in which the field φ_B is diagonal to the squtprojectile representation in which it is φ_A, which is diagonal, and φ_B is represented by a functional derivative:

$$\varphi_B(\varrho)=-\nabla_\varrho^{-2}\frac{\delta}{\delta\varphi_A(\varrho)} . \tag{33}$$

In this representation the Hamiltonian will be obtained from (32) by interchanging A and B and changing signs of derivatives. The state vector will be obtained by a quasi-Fourier transformation using (30).

One can construct a formulation in which the symmetry between target and projectile is more explicit. To do this one can pass to slightly different field variables for which the BFKL Hamiltonian becomes symmetric. The form of these new variables is clearly seen from the commutation relation (28)

$$\begin{aligned}\varphi(y,\varrho)&=\sqrt{\nabla_\varrho^2}\varphi_A(y,\varrho)\equiv T\varphi_A(y,\varrho) ,\\ \varphi^\dagger(y,\varrho)&=\sqrt{\nabla_\varrho^2}\varphi_B(y,\varrho)\equiv T\varphi_B(y,\varrho)\end{aligned} \tag{34}$$

(the operator $\sqrt{\nabla_\varrho^2}$ has a simple form in the momentum space). For them the equal rapidity commutation relation takes the form

$$\varphi(y,\varrho),\varphi^\dagger(y,\varrho')=\delta(\varrho-\varrho') . \tag{35}$$

One can also assume that the scalar product of state vectors is chosen to make φ and $\varphi^\dagger$ Hermitian conjugate to each other. In the representation in which, say, $\varphi^\dagger$ is diagonal with complex eigenvalues α, we take (up to a normalization factor)

$$\langle\Psi_1|\Psi_2\rangle=\int D\alpha D\alpha^*\Psi_1(\alpha^*)\Psi_2(\alpha)\mathrm{e}^{-\int \mathrm{d}\varrho\alpha^*(\varrho)\alpha(\varrho)} . \tag{36}$$

Then indeed

$$\begin{aligned}&\langle\Psi_1|\varphi^\dagger(\varrho)|\Psi_2\rangle\\ &=\int D\alpha D\alpha^*\Psi_1(\alpha^*)\Psi_2(\alpha)\alpha(\varrho)\mathrm{e}^{-\int \mathrm{d}\varrho'\alpha^*(\varrho')\alpha(\varrho')}\\ &=\int D\alpha D\alpha^*\,\Psi_1(\alpha^*)\Psi_2(\alpha)\left(-\frac{\delta}{\delta\alpha^*(\varrho)}\right)\mathrm{e}^{-\int \mathrm{d}\varrho'\alpha^*(\varrho')\alpha(\varrho')}\\ &=\int D\alpha D\alpha^*\Psi_2(\alpha)\mathrm{e}^{-\int \mathrm{d}\varrho'\alpha^*(\varrho')\alpha(\varrho')}\frac{\delta}{\delta\alpha^*(\varrho)}\Psi_1(\alpha^*)\\ &=\langle\varphi(\varrho)\Psi_1|\Psi_2\rangle .\end{aligned}$$

Thus the two quantized fields $\varphi(y,\varrho)$ and $\varphi^\dagger(y,\varrho)$ acquire the standard meaning of annihilation and creation operators for a pomeron at rapidity y and space points $\varrho=\{r_1,r_2\}$.

In terms of these new field variables the free action takes the form

$$\begin{aligned}S_0&=\int \mathrm{d}y\,\mathrm{d}\varrho\varphi(\varrho)T\left(\frac{\partial}{\partial y}+H\right)T^{-1}\varphi^\dagger(\varrho)\\ &\equiv\int \mathrm{d}y\,\mathrm{d}\varrho\varphi(\varrho)\left(\frac{\partial}{\partial y}+\bar H\right)\varphi^\dagger(\varrho) ,\end{aligned} \tag{37}$$

where the new Hamiltonian for the pomeron is

$$\begin{aligned}\bar H&=THT^{-1}\\ &=\frac{\bar\alpha}{2}\left(\ln p_1^2+\ln p_2^2+\sqrt{\frac{p_2^2}{p_1^2}}\ln r_{12}^2\sqrt{\frac{p_1^2}{p_2^2}}\right.\\ &\quad\left.+\sqrt{\frac{p_1^2}{p_2^2}}\ln r_{12}^2\sqrt{\frac{p_2^2}{p_1^2}}-4\psi(1)\right) .\end{aligned} \tag{38}$$

It has obviously the same eigenvalues but is Hermitian (and real). Using its hermiticity we can revert the order of operators in (37) and write S_0 in the 'normal order' form

$$S_0 = \int \mathrm{d}y\,\mathrm{d}\varrho\varphi^\dagger(\varrho)\left(-\frac{\partial}{\partial y}+\bar{H}\right)\varphi(\varrho)\,. \tag{39}$$

This form explicitly shows the symmetry between target and projectile, which is quite similar to the usual time reversal: one has to change $\varphi \leftrightarrow \varphi^\dagger$, $y \to -y$ and revert the order of all operators.

In terms of new field operators the external part of the action acquires the form

$$\begin{aligned} S_\mathrm{E} &= -\int \mathrm{d}z\left(\varphi(z)T^{-1}J_B(z)+\varphi^\dagger J_A(z)\right)\\ &\equiv -\int \mathrm{d}z\left(\varphi(z)J^\dagger(z)+\mathrm{h.c.}\right), \end{aligned} \tag{40}$$

where

$$J(z) = T^{-1}J_A(z)\,,\quad J^\dagger(z) = T^{-1}J_B(z)\,. \tag{41}$$

The interaction part becomes rather complicated, involving several operators T or their inverses:

$$\begin{aligned} S_\mathrm{I} = \frac{2\alpha_\mathrm{s}^2N_c}{\pi}\int \mathrm{d}y\frac{\mathrm{d}^2r_1\,\mathrm{d}^2r_2\,\mathrm{d}^2r_3}{r_{12}^2r_{23}^2r_{31}^2}\\ \times\left(T^{-1}\varphi^\dagger(y,\varrho_{23})T^{-1}\varphi^\dagger(y,\varrho_{13})r_{12}^4T\varphi(y,\varrho_{12})+\mathrm{h.c.}\right). \end{aligned} \tag{42}$$

With the physical meaning of the operators φ and $\varphi^\dagger$ well established and indeed standard, the analysis of the evolution becomes trivial. Let us follow it for free pomerons. Then their number is conserved and actually the only connected diagram corresponds to a single pomeron. Such a state is to be constructed as a superposition of single pomerons at different positions ϱ:

$$\Psi(y) = \int \mathrm{d}\varrho f(y,\varrho)\varphi^\dagger(\varrho)\Psi_0\,, \tag{43}$$

where Ψ_0 is the vacuum state which obeys

$$\varphi(\varrho)\Psi_0 = 0 \tag{44}$$

and is normalized to unity (we assume the Schrödinger-like picture with operators ψ and $\psi^\dagger$ at fixed rapidity). At the initial rapidity $y=0$ the pomeron wave function is determined by the external current:

$$\Psi(0) = \int \mathrm{d}\varrho\bar{J}^\dagger(\varrho)\varphi^\dagger(\varrho)\Psi_0\,, \tag{45}$$

where we recall that $\bar{J}$ is the spatial part of J. This state evolves to the final rapidity Y at which we are interested in the amplitude A_{fi} to pass to a specific final state determined by the current at $y=Y$:

$$\Psi_f = \int \mathrm{d}\varrho\bar{J}(\varrho)\varphi^\dagger(\varrho)\Psi_0\,. \tag{46}$$

One has

$$\begin{aligned} A_{fi} &= \langle\Psi_f|\Psi(Y)\rangle\\ &= \int \mathrm{d}\varrho\,\mathrm{d}\varrho'\bar{J}(\varrho)f(y,\varrho')\left\langle\Psi_0|\varphi(\varrho)\varphi^\dagger(\varrho')|\Psi_0\right\rangle\\ &= \int \mathrm{d}\varrho\bar{J}(\varrho)f(y,\varrho)\,, \end{aligned} \tag{47}$$

where we used (35) and (44).

The law which governs the evolution of the wave function $f(y,\varrho)$ follows from the general Schrödinger equation (31) and the form of the Hamiltonian $\mathcal{H}$. The free part of the latter in terms of new operators has the standard form

$$\mathcal{H}_0 = -\int \mathrm{d}\varrho\varphi^\dagger(\bar{z})\bar{H}\varphi(\bar{z})\,, \tag{48}$$

so that from (43) one immediately finds the equation

$$\frac{\partial f(y,\varrho)}{\partial y} = -\bar{H}f(y,\varrho)\,, \tag{49}$$

with a formal solution

$$f(z) = \mathrm{e}^{-\bar{H}y}f(0) = \int \mathrm{d}\varrho'\bar{g}(y,\varrho;0,\varrho')f(0)\,. \tag{50}$$

Here $\bar{g}$ is the Green function for the operator $\partial/\partial y+\bar{H}$ which can be written as an operator in the coordinate space

$$\bar{g}(y) = \theta(y)\mathrm{e}^{-\bar{H}y}. \tag{51}$$

Using this we obtain for the amplitude

$$A_{fi} = \int \mathrm{d}\varrho\,\mathrm{d}\varrho'\bar{J}(\varrho)\bar{g}(y,\varrho;0,\varrho')\bar{J}^\dagger(\varrho')\,. \tag{52}$$

Returning to the initial external sources and the Green function we reproduce the standard result

$$A_{fi} = \int \mathrm{d}\varrho\,\mathrm{d}\varrho'\bar{J}_A(\varrho)g(y,\varrho;0,\varrho')\bar{J}_B(\varrho')\,. \tag{53}$$

Indeed we have

$$\bar{g} = \left(\frac{\partial}{\partial y}+\bar{H}\right)^{-1} = T\left(\frac{\partial}{\partial y}+H\right)^{-1}T^{-1} = TgT\,. \tag{54}$$

Putting this into (53) gives (52).

4 The Schwinger–Dyson equations for the pomeron Green function

4.1 The pomeron self-mass

The pomeron self-mass operator starts and finishes with the three-pomeron vertex, which contains the operator L acting on the incoming and outgoing pomeron propagator.

As a result, the Dyson equation for the full pomeron Green function $G(z,z')$ takes the form

$$G(z,z') = g(z|z') - \int \mathrm{d}\tilde{z}_{\mathrm{inv}}\,\mathrm{d}\tilde{z}'_{\mathrm{inv}} \left[\tilde{r}_{12}^4 \tilde{\nabla}_1^2 \tilde{\nabla}_2^2 g(z,\tilde{z})\right] \times \Sigma(\tilde{z},\tilde{z}') \left[\tilde{r'}_{12}^4 \tilde{\nabla'}_1^2 \tilde{\nabla'}_2^2 G(\tilde{z}',z')\right], \quad (55)$$

where we have explicitly shown how both operators L act on pomeron propagators (the minus sign in front of the second term is due to the propagator being in fact equal to $-g$). Applying to this equation the operators L from the left and from the right, we find

$$\tilde{G}(z,z') = \tilde{g}(z,z') - \int \mathrm{d}\tilde{z}_{\mathrm{inv}}\,\mathrm{d}\tilde{z}'_{\mathrm{inv}} \tilde{g}(z,\tilde{z})\Sigma(\tilde{z},\tilde{z}')\tilde{G}(\tilde{z}',z'), \quad (56)$$

where we define

$$\tilde{G} = LGL, \quad \tilde{g} = LgL. \quad (57)$$

Equation (57) can be rewritten in an obvious operator form as

$$\tilde{G} = \tilde{g} - \tilde{g}\Sigma\tilde{G}, \quad (58)$$

which is the standard form for the Dyson equation, except for the sign.

Note that the self-mass Σ entering this equation is a conformally invariant function, due to conformal invariance of both the pomeron propagator and the three-pomeron vertex. As a result, the full Green function is also conformally invariant.

To pass to pomeron squtenergies ω we have to understand how they are related in the three-pomeron vertex. We in a standard way present

$$G(y) = \int_{a-\mathrm{i}\infty}^{a+\mathrm{i}\infty} \frac{\mathrm{d}\omega}{2\pi\mathrm{i}} \mathrm{e}^{\omega y} G(\omega), \quad (59)$$

with the inverse transform

$$G(\omega) = \int_{-\infty}^{+\infty} \mathrm{d}y \mathrm{e}^{-\omega y} G(y). \quad (60)$$

In the lowest non-trivial order the contribution to G is just a simple loop of two pomerons inserted into the pomeron propagator. Let the rapidity intervals for G, the loop, and propagators above and below the loop be Y, y'', y and y', respectively. Then, suppressing the integration over coordinates and coupling at the vertexes, we have

$$G(Y) = \int \mathrm{d}y\,\mathrm{d}y'\,\mathrm{d}y''\delta(y+y'+y''-Y) \times g(y)g_1(y'')g_2(y'')g(y'), \quad (61)$$

where g_1 and g_2 are the propagators of the two intermediate pomerons in the loop. For $G(\omega)$ we find

$$G(\omega) = \int \mathrm{d}y\,\mathrm{d}y'\,\mathrm{d}y'' \mathrm{e}^{-\omega(y+y'+y'')} g(y)g_1(y'')g_2(y'')g(y') = g(\omega)\int \mathrm{d}y \mathrm{e}^{-\omega y} g_1(y)g_2(y)g(\omega). \quad (62)$$

The integral over y can be written in the form

$$\int \mathrm{d}y \mathrm{e}^{-\omega y} \int_{a_1-\mathrm{i}\infty}^{a_1+\mathrm{i}\infty} \frac{\mathrm{d}\omega_1}{2\pi\mathrm{i}} \int_{a_2-\mathrm{i}\infty}^{a_2+\mathrm{i}\infty} \frac{\mathrm{d}\omega_2}{2\pi\mathrm{i}} \mathrm{e}^{y(\omega_1+\omega_2)} g_1(\omega_1)g_2(\omega_2). \quad (63)$$

We can always choose ω on the line

$$\mathrm{Re}(\omega-\omega_1-\omega_2) = 0.$$

Then integration over y will give

$$2\pi\delta(\mathrm{Im}(\omega-\omega_1-\omega_2)).$$

Subsequent integration over, say, ω_2 will finally give

$$\int \mathrm{d}y \mathrm{e}^{-\omega y} g_1(y)g_2(y) = \int_{a_1-\mathrm{i}\infty}^{a_1+\mathrm{i}\infty} \frac{\mathrm{d}\omega_1}{2\pi\mathrm{i}} g_1(\omega_1)g_2(\omega-\omega_2), \quad (64)$$

which result should be analytically continued to arbitrary ω. This means that in terms of energies the three-pomeron vertex formally contains

$$2\pi\mathrm{i}\delta(\omega-\omega_1-\omega_2). \quad (65)$$

That is, energies are conserved at the vertex.

Using this result, again in the lowest order, we find, explicitly showing the coordinates, $(1,2) \equiv \{r_1, r_2\}$,

$$\Sigma_\omega^{(0)}(1,2|1',2') = \frac{8\alpha_{\mathrm{s}}^4 N_c^2}{\pi^2} \int \frac{\mathrm{d}\omega_1}{2\pi\mathrm{i}} \int \frac{r_{12}\,\mathrm{d}^2 r_3}{r_{31}^2 r_{21}^2} \frac{r'_{12}\,\mathrm{d}^2 r'_3}{{r'_{31}}^2 {r'_{21}}^2} \times g_{\omega_1}(1,3|1',3')g_{\omega-\omega_1}(2,3|2',3'). \quad (66)$$

To pass to the full self-mass we have to substitute the full pomeron Green functions G for the propagators g and change one of the three-pomeron vertices into the full one Γ. In this way we obtain

$$\Sigma_\omega(1,2|1',2') = \frac{8\alpha_{\mathrm{s}}^4 N_c^2}{\pi^2} \int \frac{\mathrm{d}\omega_1}{2\pi\mathrm{i}} \int \frac{r_{12}\,\mathrm{d}^2 r_3}{r_{31}^2 r_{21}^2} \times \mathrm{d}\tau(1'',3'')\,\mathrm{d}\tau(2'',4'') \times G_{\omega_1}(1,3|1'',3'')G_{\omega-\omega_1}(2,3|2'',4'') \times \Gamma_{\omega,\omega_1}(1'',2'',3'',4''|1',2'). \quad (67)$$

In the lowest order the vertex is given by the spatial part of γ, (14):

$$\Gamma^{(0)}(1'',2'',3'',4''|1',2') = \delta^2(1''-1')\delta^2(2''-2')\delta^2(3''-4'')r_{1'2'}^2 r_{1''3''}^2 r_{2''4''}^2. \quad (68)$$

4.2 The vertex equation

As in any quantum field theory with a triple interaction the full vertex is determined by an infinite sequence of

skeleton diagrams, containing the vertex itself and full Green functions. In the lowest order one has the so-called 'three-gamma' equation. Its explicit form is rather complicated due to the large number of variables. From the corresponding Feynman diagram we find in terms of gluon coordinates

$$\begin{aligned}\Gamma_{\omega,\omega'}(1',2',3',4'|1,2) &= \Gamma^{(0)}(1',2',3',4'|1,2)\\ &- \frac{8\alpha_s^4 N_c^2}{\pi^2}\int\frac{\mathrm{d}\omega_1}{2\pi\mathrm{i}}\int \mathrm{d}\tau(1'',2'')\,\mathrm{d}\tau(3'',4'')\,\mathrm{d}\tau(3''',4''')\\ &\times \mathrm{d}\tau(\tilde1,\tilde2)\,\mathrm{d}\tau(\tilde1',\tilde2')\,\mathrm{d}\tau(1''',2''')L(1',3')\\ &\times \Gamma_{\omega',\omega_1}(1',3'|1'',2'',3'',4'')G_{\omega_1}(1'',2''|1''',2'\,prime')\\ &\times G_{\omega'-\omega_1}(3'',4''|3''',4''')\Gamma_{\omega-\omega_1,\omega-\omega'}(2',4',3''',4'''|\tilde1,\tilde2)\\ &\times L(\tilde1,\tilde2)G_{\omega-\omega_1}(\tilde1,\tilde2|\tilde1',\tilde2')\Gamma_{\omega,\omega_1}(1''',2''',\tilde1',\tilde2'|1,2)\,.\end{aligned} \tag{69}$$

In the shorthand notation $\varrho=\{r_1,r_2\}$ and correspondingly

$$G_\omega(1,2)\equiv G_\omega(\varrho_1,\varrho_2)\equiv G(r_1^{(1)},r_2^{(1)}r_1^{(2)}r_2^{(2)})\,,$$

the equation for the vertex part can be rewritten in a more compact form

$$\begin{aligned}\Gamma_{\omega,\omega'}(1',2'|1) &= \Gamma^{(0)}(1',2'|1)\\ &- \frac{8\alpha_s^4 N_c^2}{\pi^2}\int\frac{\mathrm{d}\omega_1}{2\pi\mathrm{i}}\int \mathrm{d}\tau(1'')\,\mathrm{d}\tau(2'')\,\mathrm{d}\tau(3'')\,\mathrm{d}\tau(4'')\\ &\times \mathrm{d}\tau(5'')\,\mathrm{d}\tau(6'')L(1')\Gamma_{\omega',\omega_1}(1'|1'',2'')G_{\omega_1}(1''|6'')\\ &\times G_{\omega'-\omega_1}(2''|3'')\Gamma_{\omega-\omega_1,\omega-\omega'}(2',3''|4'')L(4'')\\ &\times G_{\omega-\omega_1}(4''|5'')\Gamma_{\omega,\omega_1}(5'',6''|1)\,.\end{aligned} \tag{70}$$

5 Conformal (Möbius) invariance

5.1 Conformal basis

The basic building blocks of the pomeron perturbation theory – the propagator g, the bare three-pomeron vertex γ and the integration volume $\mathrm{d}\tau$ – are conformally invariant. As a result also the full Green function G, self-mass Σ and vertex Γ are conformally invariant functions of their arguments. It seems profitable to use this property to simplify the Schwinger–Dyson equations. To do this we have to study a general form for the conformally invariant functions for the transitions pomeron → pomeron (squttwo-point functions) and pomeron → 2 pomerons (squtthree-point functions). This can be achieved by expanding these functions in the conformal basis formed by the functions (in complex notation) [21]

$$E_\mu(\varrho)=E_\mu(r_1,r_2)=\left(\frac{r_{12}}{r_{10}r_{20}}\right)^{\frac{1-n}{2}+\mathrm{i}\nu}\left(\frac{r_{12}^*}{r_{10}^*r_{20}^*}\right)^{\frac{1+n}{2}+\mathrm{i}\nu}, \tag{71}$$

where $\mu=\{n,\nu,r_0\}=\{h,r_0\}$ with n integer, ν real, and the two-dimensional transverse r_0 enumerate the basis.

The functions $E_\mu(\varrho)$ are the proper functions of the operator L:

$$LE_\mu(\varrho)=l_\mu E_\mu(\varrho)\,,\quad l_\mu=\frac{4\pi^8}{a_{n+1,\nu}a_{n-1,\nu}}\,, \tag{72}$$

where

$$a_{n,\nu}\equiv a_\mu=\frac{\pi^4}{2}\frac{1}{\nu^2+n^2/4}\,. \tag{73}$$

They form a complete system:

$$r_{12}^4\delta(\varrho-\varrho')=\frac12\sum_\mu E_\mu(\varrho)E_\mu^*(\varrho')\,, \tag{74}$$

where we use the notation

$$\sum_\mu=\sum_{n=-\infty}^{\infty}\int\mathrm{d}\nu\frac{1}{a_{n,\nu}}\int\mathrm{d}^2r_0\,, \tag{75}$$

and we satisfy the orthogonality relation

$$\begin{aligned}\int\mathrm{d}\tau E_\mu(\varrho)E_{\mu'}^*(\varrho') &= a_{n,\nu}\delta_{nn'}\delta(\nu-\nu')\delta^2(r_{00'})\\ &+ b_{n\nu}\delta_{n,-n'}\delta(\nu+\nu')|r_{00'}|^{-2-4\mathrm{i}\nu}\left(\frac{r_{00'}}{r_{00'}^*}\right)^n.\end{aligned} \tag{76}$$

The coefficients $b_{n\nu}$ are given by the formula

$$b_{n\nu}=\pi^3\frac{2^{4\mathrm{i}\nu}}{-\mathrm{i}\nu+|n|/2}\frac{\Gamma(-\mathrm{i}\nu+(1+|n|)/2)\Gamma(\mathrm{i}\nu+|n|/2)}{\Gamma(\mathrm{i}\nu+(1+|n|)/2)\Gamma(-\mathrm{i}\nu+|n|/2)}\,. \tag{77}$$

The functions E_μ are not linearly independent. In fact

$$E_{-n,-\nu,r_0}(\varrho)=\frac{b_{n\nu}}{a_{n\nu}}\int\mathrm{d}^2r_0'|r_{00'}|^{-2+4\mathrm{i}\nu}\left(\frac{r_{00'}^*}{r_{00'}}\right)^n E_{n,\nu,r_0'}(\varrho)\,. \tag{78}$$

Presenting a function $f(\varrho)$ by

$$f(\varrho)=\int\mathrm{d}\varrho'\delta(\varrho-\varrho')f(\varrho') \tag{79}$$

and using the completeness of (74) one gets

$$f(\varrho)=\frac12\int\mathrm{d}\tau'\sum_\mu E_\mu(\varrho)E_\mu^*(\varrho')f(\varrho')=\frac12\sum_\mu E_\mu(\varrho)f_\mu\,, \tag{80}$$

where

$$f_\mu=\int\mathrm{d}\tau E_\mu^*(\varrho)f(\varrho)\,. \tag{81}$$

This gives the standard expansion in the whole overcomplete basis.

However it seems possible to limit oneself to an independent part of this basis. One possibility is to take a restricted basis with $\nu>0$ ($\nu<0$). We denote this restriction

by $\mu>0$ ($\mu<0$). In fact we may split the integration over ν in (80) into two parts:

$$f(\varrho)=\frac{1}{2}\sum_{\mu>0}E_\mu(\varrho)f_\mu+\frac{1}{2}\sum_{\mu<0}E_\mu(\varrho)f_\mu\,, \tag{82}$$

and, say, in the second term express the eigenfunctions E_μ with $\mu<0$ via those with $\mu>0$ using (78) to obtain for the second term

$$\frac{1}{2}\sum_{\mu<0}f_\mu\frac{b_{-n-\nu}}{a_{-n-\nu}}\int \mathrm{d}^2r_0'|r_{00'}|^{-2-4\mathrm{i}\nu}\left(\frac{r^*_{00'}}{r_{00'}}\right)^{-n}E_{-n,-\nu,r_0'}(\varrho)\,. \tag{83}$$

Changing the summation and integration variables $n\to -n$ and $\nu\to-\nu$ we get for this term

$$\frac{1}{2}\sum_{\mu>0}f_{-n,-\nu,r_0}\frac{b_{n\nu}}{a_{n\nu}}\int \mathrm{d}^2r_0'|r_{00'}|^{-2+4\mathrm{i}\nu}\left(\frac{r^*_{00'}}{r_{00'}}\right)^{n}E_{n,\nu,r_0'}(\varrho)\,. \tag{84}$$

Interchanging also r_0 and r_0' integrations we finally find for it

$$\frac{1}{2}\sum_{\mu>0}E_\mu(\varrho)\bar f_\mu\,, \tag{85}$$

where

$$\bar f_\mu=\frac{b_{n\nu}}{a_{n\nu}}\int \mathrm{d}^2r_0' f_{-n,-\nu,r_0'}|r_{0'0}|^{-2+4\mathrm{i}\nu}\left(\frac{r^*_{0'0}}{r_{0'0}}\right)^{n}. \tag{86}$$

Summing this with the first term in (82) we get the desired expansion in the states with $\mu>0$:

$$f(\varrho)=\frac{1}{2}\sum_{\mu>0}E_\mu(\varrho)(f_\mu+\bar f_\mu)\equiv\sum_{\mu>0}E_\mu(\varrho)\lambda_\mu\,. \tag{87}$$

Integrating this with $E^*_{\mu>0}(\varrho)$ and using (76), we find

$$\lambda_\mu=\int \mathrm{d}\tau E^*_\mu(\varrho)f(\varrho)\,. \tag{88}$$

Putting this into (87) we find

$$f(\varrho)=\sum_{\mu>0}E_\mu(\varrho)\int \mathrm{d}\tau' E^*_\mu(\varrho')f(\varrho')\,, \tag{89}$$

which means that one also has a completeness relation for half of the basis with $\nu>0$:

$$r_{12}^4\delta(\varrho-\varrho')=\sum_{\mu>0}E_\mu(\varrho)E^*_\mu(\varrho')\,. \tag{90}$$

Obviously the same property is valid for the second half of the basis with $\nu<0$.

5.2 Two-point functions

We have to deal with a conformally invariant function $A(1|1')$, where, as introduced in the previous sections, the arguments refer to pairs of the pomeron coordinates. Using half of the conformal basis we present A by

$$\begin{aligned}A(1|1')&=\sum_{\mu,\mu'>0}E_\mu(1)E^*_{\mu'}(1')A_{\mu\mu'}\\&=\sum_{\mu>0}\sum_{\mu'<0}E_\mu(1)E_{\mu'}(1')A_{\mu\mu'}\,,\end{aligned} \tag{91}$$

where we use

$$E^*_{n,\nu,r_0}(\varrho)=E_{-n,-\nu,r_0}(\varrho)\,. \tag{92}$$

Our aim is to see which properties $A_{\mu,\mu'}$ should have for the function $A(1,1')$ to be conformally invariant. Since $E_{n,\nu,r_0}(r_1,r_2)=E_{n,\nu,r_0+a}(r_1+a,r_2+a)$ the translational invariance requires $A_{\mu\mu'}$ to depend only on the difference $r_0-r_0'\equiv r_{00'}$. Under inversion the expression to be summed over $\mu>0$ and $\mu'<0$ changes as follows:

$$A_{\mu\mu'}(r_{00'})\to r_0^{-1-n+2\mathrm{i}\nu}{r_0'}^{-1-n'+2\mathrm{i}\nu'}(\text{a.f.})A_{\mu\mu'}\left(\frac{r_{00'}}{r_0r_0'}\right). \tag{93}$$

Here and in the following (a.f.) means 'antiholomorhic factor'; that is, the complex conjugate of the preceding factor. Invariance under inversion requires that

$$A_{\mu\mu'}\left(\frac{r_{00'}}{r_0r_0'}\right)=r_0^{1+n-2\mathrm{i}\nu}{r_0'}^{1+n'-2\mathrm{i}\nu'}(\text{a.f.})A_{\mu\mu'}(r_{00'})\,. \tag{94}$$

However the left-hand side only depends on the product r_0r_0', so in the right-hand side we are obliged to have either $n=n'$ and $\nu=\nu'$ or $A_{\mu\mu'}(r_{00'})\propto\delta(r_{00'})$. Since ν and ν' have opposite signs, the first alternative cannot be realized. With $A_{\mu\mu'}(r_{00'})\propto\delta(r_{00'})$. We find that $n+n'=0$ and $\nu+\nu'=0$, so that

$$A_{\mu\mu'}=\delta_{n,-n'}\delta(\nu+\nu')\delta(r_{00'})a_{n\nu}A_{n\nu}\equiv\delta_{\mu,\bar\mu'}A_\mu\,, \tag{95}$$

where $\bar\mu=\mu(n\to-n,\nu\to-\nu)$, and we defined $A_\mu\equiv A_{n\nu}$ with the factor $a_{n\nu}$ separated for convenience. As a result the double sum in (91) transforms into a single one:

$$A(1|1')=\sum_{\mu>0}E_\mu(1)E_{\bar\mu}(1')A_{n\nu}=\sum_{\mu>0}E_\mu(1)E^*_\mu(1')A_{n\nu}\,, \tag{96}$$

where $\bar\mu=\mu(n\to-n,\nu\to-\nu)$. A similar form with summation over $\mu<0$ can be obtained in the same manner. Representation (96) or a similar one with a sum over $\mu<0$ is valid for any conformally invariant two-point function. Note that taking an average of the sums over $\mu>0$ and $\mu<0$ one obtains a similar representation in terms of the whole overcomplete basis, which is in a standard way used for the BFKL Green function $g(z,z')$.

Now suppose we have conformally invariant functions $B(1|1')$ and $C(1,1')$ and form a conformally invariant integral:

$$A(1|1')=\int \mathrm{d}\tau'' B(1|1'')C(1''|1')\,. \tag{97}$$

Each of the three functions A, B and C has the representation (96) with conformal coefficients A_μ, B_μ and C_μ. Doing the integration with the help of the orthonormalization properties of the basis functions E_μ with $\mu > 0$, we find

$$A(1|1') = \sum_{\mu>0} E_\mu(1) E^*_\mu(1') B_\mu C_\mu = \sum_{\mu>0} E_\mu(1) E^*_\mu(1') A_\mu \,, \tag{98}$$

which means that in the conformal representation (96)

$$A_\mu = B_\mu C_\mu \,. \tag{99}$$

Applying this result to the Dyson equation (58) in the ω representation, we immediately find

$$\tilde{G}_{\omega\mu} = \tilde{g}_{\omega\mu} - \tilde{g}_{\omega\mu} \Sigma_{\omega\mu} \tilde{G}_{\omega\mu} \,. \tag{100}$$

So, both the energy ω and conformal quantum numbers μ of the pomeron are conserved in the interaction and its full Green function in the conformal basis is trivially expressed via its self-mass:

$$\tilde{G}_{\omega\mu} = \frac{1}{1/\tilde{g}_{\omega\mu} + \Sigma_{\omega\mu}} \quad \text{or} \quad G_{\omega\mu} = \frac{1}{1/g_{\omega\mu} + l_\mu^2 \Sigma_{\omega\mu}} \,. \tag{101}$$

In (101) the free pomeron Green function g in the conformal basis is given by

$$g_{\omega\mu} = \frac{2}{l_{n\nu}} \frac{1}{\omega - \omega_{n\nu}} \,. \tag{102}$$

5.3 Three-point functions

For a three-point function the expansion similar to (96) reads

$$\Gamma(1|2,3) = \sum_{\mu_1,\mu_2\mu_3>0} E_{\mu_1}(1) E^*_{\mu_2}(2) E^*_{\mu_3}(3) \Gamma_{\mu_1|\mu_2\mu_3} \,, \tag{103}$$

where the intermediate c.m. coordinates are R_1, R_2 and R_3. Conformal invariance allows one to determine the dependence on them of $\Gamma_{\mu_1|\mu_2\mu_3}$. Translational invariance requires $\Gamma_{\mu_1|\mu_2\mu_3}$ to depend only on the differences R_{ik} and from the scale invariance such a dependence should be a power one. So, we seek

$$\Gamma_{\mu_1|\mu_2\mu_3} = R_{12}^{\alpha_{12}} R_{23}^{\alpha_{23}} R_{31}^{\alpha_{31}} (\text{a.f.}) \Gamma_{n_1\nu_1|n_2\nu_2 n_3\nu_3}, \,. \tag{104}$$

After inversion we find an extra factor in the sum (102):

$$R_1^{-2-\alpha_{12}-\alpha_{31}+1-n_1+2\mathrm{i}\nu_1} (\text{a.f.}) \mathrm{e}^{-\mathrm{i}n_1\pi} \tag{105}$$

times two similar factors which are obtained from (104) by cyclic permutations of 123 and conjugation. Invariance requires that each power is zero and that the sum $n_1 + n_2 + n_3$ be even. We get six equations to determine the α. Their solution gives

$$\begin{aligned}
\alpha_{12} &= -\frac{1}{2} + \frac{1}{2}(n_2 - n_1 - n_3) + \mathrm{i}(\nu_1 - \nu_2 + \nu_3) \,, \\
\alpha_{23} &= -\frac{1}{2} + \frac{1}{2}(n_1 + n_2 + n_3) - \mathrm{i}(\nu_1 + \nu_2 + \nu_3) \,, \\
\alpha_{31} &= -\frac{1}{2} + \frac{1}{2}(n_3 - n_1 - n_2) + \mathrm{i}(\nu_1 + \nu_2 - \nu_3)
\end{aligned} \tag{106}$$

and similar expressions for the powers in the antiholomorhic factor of the $\tilde{\alpha}$ with $n_i \to -n_i$.

For the free vertex $\Gamma^{(0)}$ given by (68), we have

$$\Gamma^{(0)}_{\mu_1|\mu_2\mu_3} = V_{\bar{\mu}_1\mu_2\mu_3} \,, \tag{107}$$

where again $\bar{\mu} = \mu(n \to -n, \nu \to -\nu)$ and the vertex $V_{\mu_1\mu_2\mu_3}$ has been introduced by Korchemsky [22]:

$$\begin{aligned}
V_{\mu_1\mu_2\mu_3} &= \int \frac{\mathrm{d}^2 r_1 \, \mathrm{d}^2 r_2 \, \mathrm{d}^2 r_3}{r_{12}^2 r_{23}^2 r_{31}^2} \\
&\quad \times E_{\mu_1}(r_1, r_2) E_{\mu_2}(r_2, r_3) E_{\mu_3}(r_3, r_1) \\
&= \Omega(h_1, h_2, h_3) \prod_{i<j} r_{0_i 0_j}^{-\Delta_{ij}} r^{*\,-\bar{\Delta}_{ij}}_{0_i 0_j} \,,
\end{aligned} \tag{108}$$

with $\Delta_{12} = h_1 + h_2 - h_3$ etc.

6 Σ and Γ in the conformal basis

To illustrate the simplifications introduced by the transition to the conformal basis in this section we study the pomeron self-mass and triple interaction vertex in this basis.

In a shorthand notation for the gluon coordinates $1 = (r_1, \bar{r}_1)$ the pomeron self-mass is given by

$$\begin{aligned}
\Sigma_\omega(1|1') = \frac{8\alpha_\mathrm{s}^4 N_c^2}{\pi^2} \int \frac{\mathrm{d}\omega_1}{2\pi\mathrm{i}} \int \mathrm{d}\tau(2) \, \mathrm{d}\tau(3) \, \mathrm{d}\tau(2') \, \mathrm{d}\tau(3') \\
\times \Gamma^{(0)}(1|2,3) G_{\omega_1}(2|2') G_{\omega-\omega_1}(3|3') \Gamma_{\omega,\omega_1}(2',3'|1') \,.
\end{aligned} \tag{109}$$

The actual number of integrations is in fact smaller due to the δ-functions in the conformal vertex $\Gamma^{(0)}$ defined by (68). We expand both the Green function and the vertexes in the conformal basis with $\mu > 0$ (see (96) and (103)). In the following, we also suppress the ω-dependence to economize on subindexes. We have

$$G(2|2') = \sum_{\mu>0} G_\mu E_\mu(2) E^*_\mu(2') \,, \tag{110}$$

$$G(3|3') = \sum_{\mu'>0} G_{\mu'} E_{\mu'}(3) E^*_{\mu'}(3') \,, \tag{111}$$

$$\Gamma^{(0)}(1|2,3) = \sum_{\mu_1,\mu_2,\mu_3>0} \Gamma^{(0)}_{\mu_1|\mu_2,\mu_3} E_{\mu_1}(1) E^*_{\mu_2}(2) E^*_{\mu_3}(3) \,, \tag{112}$$

$$\Gamma(2',3'|1') = \sum_{\mu'_1,\mu'_2,\mu'_3>0} \Gamma_{\mu'_2,\mu'_3|\mu'_1} E^*_{\mu'_1}(1') E_{\mu'_2}(2') E_{\mu'_3}(3') \,. \tag{113}$$

Integrations over the gluon coordinates are done with the help of (76) and give the product of δ-functions

$$a_\mu a_{\mu'} \delta_{\mu\mu_2} \delta_{\mu\mu'_{2'}} \delta_{\mu'\mu_3} \delta_{\mu'\mu_{3'}} .$$

So, we find

$$\begin{aligned} \Sigma(1,2|1',2') &= \frac{8\alpha_s^4 N_c^2}{\pi^2} \int \frac{\mathrm{d}\omega_1}{2\pi\mathrm{i}} \\ &\times \sum_{\mu_1,\mu'_1,\mu,\mu'} \Gamma^{(0)}_{\mu_1|\mu,\mu'} G_\mu G_{\mu'} \Gamma_{\mu,\mu'|\mu'_1} E_{\mu_1}(1) E^*_{\mu'_1}(1') \\ &= \sum_{\mu_1,\mu'_1} \Sigma_{\mu_1\mu'_1} E_{\mu_1}(1) E^*_{\mu'_1}(1') , \end{aligned} \tag{114}$$

where

$$\Sigma_{\mu_1\mu'_1} = \frac{8\alpha_s^4 N_c^2}{\pi^2} \int \frac{\mathrm{d}\omega_1}{2\pi\mathrm{i}} \sum_{\mu,\mu'} \Gamma^{(0)}_{\mu_1|\mu,\mu'} G_\mu G_{\mu'} \Gamma_{\mu,\mu'|\mu'_1} , \tag{115}$$

and the suppressed dependence on ω is obvious from its conservation at the vertexes. Thus we have found for $\Sigma(1,2|1',2')$ an expansion in the conformal basis (91). From its conformal invariance it follows that

$$\Sigma_{\mu_1\mu'_1} = \delta_{\mu_1\mu'_1} \Sigma_{\mu_1} , \tag{116}$$

where Σ_μ is the desired pomeron self-mass in the conformal representation. It can be found from (115) after summation over μ'_1. This gives

$$\Sigma_\mu = \frac{8\alpha_s^4 N_c^2}{\pi^2} \int \frac{\mathrm{d}\omega_1}{2\pi\mathrm{i}} \sum_{\mu_1,\mu_2,\mu_3} \Gamma^{(0)}_{\mu|\mu_1,\mu_2} G_{\mu_1} G_{\mu_2} \Gamma_{\mu_1,\mu_2|\mu_3} . \tag{117}$$

The sum over μ_i, $i = 1,2,3$ includes integrations over the three c.m. coordinates R_i on which only the vertexes depend. We get an integral depending on the four conformal weights:

$$\begin{aligned} I_{h|h_1,h_2|h_3} &= \int \mathrm{d}^2R_1 \, \mathrm{d}^2R_2 \, \mathrm{d}^2R_3 R_{01}^{\alpha_{01}} R_{02}^{\alpha_{02}} \\ &\times R_{12}^{\alpha_{12}} R_{31}^{\alpha_{31}} R_{32}^{\alpha_{32}} R_{12}^{\alpha_{21}} (\text{a.f.}) . \end{aligned} \tag{118}$$

Here R_0 is arbitrary since the integral is independent of it. The powers are given by

$$\begin{aligned} \alpha_{01} &= -\frac{1}{2} + \frac{1}{2}(n_1 - n_2 - n) + \mathrm{i}(\nu_2 - \nu_1 - \nu) , \\ \alpha_{12} &= -\frac{1}{2} + \frac{1}{2}(n_1 + n_2 + n) - \mathrm{i}(\nu_2 + \nu_1 + \nu) , \\ \alpha_{21} &= -\frac{1}{2} - \frac{1}{2}(n_1 + n_2 + n_3) + \mathrm{i}(\nu_2 + \nu_1 + \nu_3) , \\ \alpha_{31} &= -\frac{1}{2} + \frac{1}{2}(n_2 - n_1 + n_3) + \mathrm{i}(\nu_1 - \nu_2 - \nu_3) \end{aligned}$$

and α_{02} and α_{32} are obtained from α_{01} and α_{31} by interchanging $1 \leftrightarrow 2$. The integral (118) is convergent both in the infrared and ultraviolet. However, its calculation does not look simple. Once this integral is known, the self-mass in the conformal basis is given by a sum over three conformal weights:

$$\begin{aligned} \Sigma_{\omega,h}(\omega) &= \frac{8\alpha_s^4 N_c^2}{\pi^2} \int \frac{\mathrm{d}\omega_1}{2\pi\mathrm{i}} \sum_{h_1,h_2,h_3} I_{h|h_1,h_2|h_3} \Omega_{\bar{h},h_1,h_2} \\ &\times G_{h_1}(\omega_1) G_{h_2}(\omega - \omega_1) \Gamma_{\bar{h}_1,\bar{h}_2|h_3}(\omega,\omega_1) . \end{aligned} \tag{119}$$

Here we made explicit the ω-dependence introducing it in the arguments; Ω is the Korchemski vertex (107), $\Gamma_{\bar{h}_1,\bar{h}_2|h_3}$ is defined by (104) and $\sum_h$ is given by (74) without integration over r_0.

In the same manner one can obtain expressions for the vertex part Γ. We shall limit ourselves to the 'three-gamma' approximation, (79). As before we expand the vertex parts and Green functions in the conformal basis. Suppressing again the ω-dependence, we have

$$\begin{aligned} \Gamma(1'|1'',2'') &= \sum_{\mu_3,\mu'_3,\mu''_3>0} \Gamma_{\mu_3|\mu'_3,\mu''_3} E_{\mu_3}(1') E^*_{\mu'_3}(1'') E^*_{\mu''_3}(2'') , \\ \Gamma(2',3''|4'') &= \sum_{\mu_2,\mu'_2,\mu''_2>0} \Gamma_{\mu_2\mu'_2|\mu''_2} E^*_{\mu''_2}(4'') E_{\mu_2}(2') E_{\mu'_2}(3'') , \\ \Gamma(5'',6''|1) &= \sum_{\mu_1,\mu'_1,\mu''_1>0} \Gamma_{\mu'_1\mu''_1|\mu_1} E^*_{\mu_1}(1) E_{\mu'_1}(5'') E_{\mu''_1}(6'') , \\ G(1''|6'') &= \sum_{\mu_4>0} G_{\mu_4} E_{\mu_4}(1'') E^*_{\mu_4}(6'') , \\ G(2''|3'') &= \sum_{\mu_5>0} G_{\mu_5} E_{\mu_5}(2'') E^*_{\mu_5}(3'') , \\ G(4''|5'') &= \sum_{\mu_6>0} G_{\mu_6} E_{\mu_6}(4'') E^*_{\mu_6}(5'') . \end{aligned}$$

Integrations over the double primed coordinates will give six δ-functions in μ. So finally we are left with six summations over μ_i, $i = 1,\dots,6$. The result can be presented in the form (with ω-dependence suppressed)

$$\Gamma(1',2'|1) = \sum_{\mu_1,\mu_2,\mu_3>0} \Gamma_{\mu_2,\mu_3|\mu_1} E^*_{\mu_1}(1) E_{\mu_2}(1') E_{\mu_3}(2') , \tag{120}$$

where

$$\begin{aligned} \Gamma_{\mu_2,\mu_3|\mu_1} &= \Gamma^{(0)}_{\mu_2,\mu_3|\mu_1} - \frac{8\alpha_s^4 N_c^2}{\pi^2} \int \frac{\mathrm{d}\omega_1}{2\pi\mathrm{i}} l_{\mu_3} \\ &\times \sum_{\mu_4,\mu_5,\mu_6>0} l_{\mu_6} \Gamma_{\mu_3|\mu_4,\mu_5} \Gamma_{\mu_2,\mu_5|\mu_6} \Gamma_{\mu_6,\mu_4|\mu_1} G_{\mu_4} G_{\mu_5} G_{\mu_6} \end{aligned} \tag{121}$$

is the desired vertex part in the conformal basis. Its dependence on the c.m. coordinates R_i, $i = 1,2,3$, is determined according to (104). The bare vertex $\Gamma^{(0)}_{\mu_2,\mu_3|\mu_1}$ is here given by the formula analogous to (107).

In (121) summations over μ_i, $i = 4,5,6$, include integrations over the c.m. coordinates R_i, $i = 4,5,6$. In the integrand the R-dependence comes only from the vertex parts

and in its turn is defined by (104). Thus the expression

$$I_{h_1h_2h_3|h_4h_5h_6} = R_{12}^{-\alpha_{12}} R_{13}^{-\alpha_{13}} R_{23}^{-\alpha_{23}} \times \int \mathrm{d}R_4\,\mathrm{d}R_5\,\mathrm{d}R_6 R_{34}^{\alpha_{34}} R_{35}^{\alpha_{35}} R_{45}^{\alpha_{45}} R_{14}^{\alpha_{14}} \times R_{16}^{\alpha_{16}} R_{46}^{\alpha_{46}} R_{25}^{\alpha_{25}} R_{26}^{\alpha_{26}} R_{56}^{\alpha_{56}}(\text{a.f.})\,, \quad (122)$$

where all α are determined by formulas similar to (106) (see Appendix B), is independent of the c.m. coordinates R_i, $i = 1, 2, 3$ and depends only on the conformal weights. So the part of the vertex depending on conformal weights will satisfy the equation (with ω-dependence restored in the arguments)

$$\Gamma_{h_2,h_3|h_1}(\omega,\omega') = \Omega(h_1,\bar{h}_2,\bar{h}_3) - \frac{8\alpha_s^4 N_c^2}{\pi^2}\int \frac{\mathrm{d}\omega_1}{2\pi\mathrm{i}} l_{h_3} \times \sum_{h_4,h_5,h_6>0} l_{h_6} I_{h_1h_2h_3|h_4h_5h_6} \Gamma_{h_3|h_4,h_5}(\omega',\omega_1) \times \Gamma_{h_2,h_5|h_6}(\omega-\omega_1,\omega-\omega')\,\Gamma_{h_6,h_4|h_1}(\omega,\omega_1) \times G_{h_4}(\omega_1) G_{h_5}(\omega'-\omega_1)\,G_{h_6}(\omega-\omega_1)\,. \quad (123)$$

7 Conclusions

We have presented a formalism which allows one to study the interaction of pomerons in the QCD with $N_c \to \infty$ using the standard methods of quantum field theory. In particular, we constructed the Schwinger–Dyson equations which sum the diagrams for the full pomeron Green function (the "enhanced" graphs in the terminology of the old Regge–Gribov local supercritical pomeron model) and carry information of the 'physical' pomeron as compared to the 'bare' one.

Conformal symmetry of the theory leads to certain simplifications. As a result we obtain a picture very similar to the old Gribov local supercritical pomeron. The difference is reduced to an (infinite) number of pomerons with varying $n = 0, \pm2, \pm4, \ldots$ and the more complicated form of the 'energy' $\omega_{n\nu}$ as a function of ν, which plays the role of the pomeron momentum in the old theory, and of the bare triple pomeron vertex, which now depends both on n and ν. If, however, one selects the supercritical pomeron with $n = 0$ and small values of ν, the formal similarity becomes almost complete, since the energy becomes a quadratic function of ν and the bare vertex then reduces to a well-know constant [22]. Unfortunately, with this similarity also the problems of the old theory, mentioned in the Introduction, return together with the question of the internal consistency of the model. At present we do not know the answer to this question and leave it for future studies.

Appendix A: Color factors

Let the color wave function of a pair of gluons be $|ab\rangle$, where $a, b = 1, \ldots, N_c^2 - 1$. Then the vacuum color state is obviously

$$|0\rangle = \frac{1}{\sqrt{N_c^2-1}}\sum_a |aa\rangle = P|ab\rangle\,, \quad (A.1)$$

where

$$P = \frac{1}{\sqrt{N_c^2-1}}\delta_{ab} \quad (A.2)$$

is the projector onto the vacuum color state.

The color structure of the vertex for the transition $2 \to 4$ reggeized gluons, with initial and final color variables $a_1, b_1 \to a_2, b_2, a_3, b_3$ is given by the expression

$$V_c = f^{a_1a_2c} f^{cb_2d} f^{da_3e} f^{eb_3b_1}\,. \quad (A.3)$$

We want the projection of this color vertex onto the three vacuum color states formed by the gluons with colors a_1b_1, a_2b_2 and a_3b_3. Applying the three corresponding projectors P_1, P_2 and P_3, we obtain

$$P_2P_3V_cP_1 = \frac{1}{(N_c^2-1)^{3/2}} f^{a_1a_2c} f^{ca_2d} f^{da_3e} f^{ea_3a_1} = \frac{N_c^2}{(N_c^2-1)^{3/2}}\delta_{a_1d}\delta_{a_1d} = \frac{N_c^2}{\sqrt{N_c^2-1}} \simeq N_c\,. \quad (A.4)$$

Note, however, that the quarks quark loop which represents the external source has as its color factor $\delta_{ab} = \sqrt{N_c^2-1}P \simeq N_cP$, so that each external source contributes a factor N_c.

Appendix B: Powers in (122)

We have

$$\begin{aligned}
\alpha_{12} &= -\frac{1}{2} + \frac{1}{2}(n_1+n_3-n_2) + \mathrm{i}(\nu_2-\nu_3-\nu_1)\,,\\
\alpha_{13} &= -\frac{1}{2} + \frac{1}{2}(n_1+n_2-n_3) + \mathrm{i}(\nu_3-\nu_2-\nu_1)\,,\\
\alpha_{23} &= -\frac{1}{2} - \frac{1}{2}(n_1+n_2+n_3) + \mathrm{i}(\nu_1+\nu_2+\nu_3)\,,\\
\alpha_{34} &= -\frac{1}{2} + \frac{1}{2}(n_4-n_5-n_3) + \mathrm{i}(\nu_5-\nu_4+\nu_3)\,,\\
\alpha_{35} &= -\frac{1}{2} + \frac{1}{2}(n_5-n_4-n_3) + \mathrm{i}(\nu_4-\nu_5+\nu_3)\,,\\
\alpha_{45} &= -\frac{1}{2} + \frac{1}{2}(n_4+n_5+n_3) - \mathrm{i}(\nu_5+\nu_4+\nu_3)\,,\\
\alpha_{16} &= -\frac{1}{2} + \frac{1}{2}(n_1+n_4-n_6) + \mathrm{i}(\nu_6-\nu_4+\nu_1)\,,\\
\alpha_{14} &= -\frac{1}{2} + \frac{1}{2}(n_1+n_6-n_4) + \mathrm{i}(\nu_4-\nu_6+\nu_1)\,,\\
\alpha_{46} &= -\frac{1}{2} - \frac{1}{2}(n_1+n_4+n_6) + \mathrm{i}(\nu_6+\nu_4+\nu_1)\,,\\
\alpha_{26} &= -\frac{1}{2} + \frac{1}{2}(n_2+n_5-n_6) + \mathrm{i}(\nu_6-\nu_5-\nu_2)\,,
\end{aligned}$$

$$\alpha_{25} = -\frac{1}{2} + \frac{1}{2}(n_2 + n_6 - n_5) + \mathrm{i}(\nu_5 - \nu_6 - \nu_2)\,,$$
$$\alpha_{56} = -\frac{1}{2} - \frac{1}{2}(n_2 + n_5 + n_6) + \mathrm{i}(\nu_6 + \nu_5 + \nu_2)\,.$$

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